

a secular yaw drift equal to the product of the misalignment angle and the yaw-roll quadrature covariance, divided by the center frequency of oscillation. (Actually, scale factor errors can in general produce additional secular drift in such an environment, although these errors were dismissed in Ref. 3.) The same general formulation (statistical moments involving the components of a vector cross product) lends itself to evaluation of computational drifts arising from a Runge-Kutta integration algorithm. This calls for establishing the covariance of a Gaussian waveform with the error in an n th-order polynomial fitted to that waveform. Secular drifts produced would be characterized as resulting from combined processing of angular information from different channels while commutativity holds only in the infinitesimal limit.

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Comment on "An Empirical Expression for Drag Coefficients of Cones at Supersonic Speeds"

LEONARD W. SCHWARTZ*

Lockheed Missiles & Space Company,
Sunnyvale, Calif.

HILL¹ presents a curve fit for the forebody drag coefficient of sharp cones at zero yaw. The forebody drag coefficient is identically equal to the pressure coefficient for sharp cones when the Mach number is high enough to permit shock

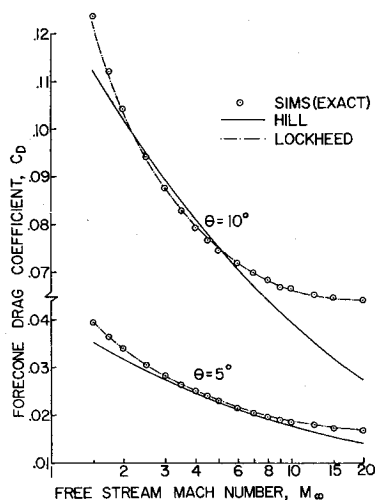


Fig. 1 Variation of forecone drag coefficient with Mach number for sharp cones of half-angle 5° and 10°.

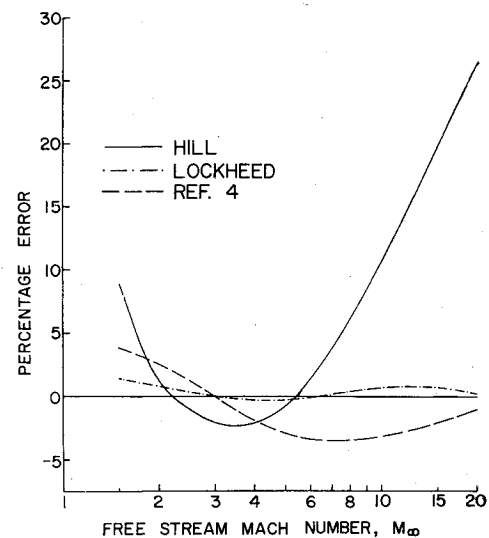


Fig. 2 Percentage error of various theories in predicting forecone drag coefficients for a half-cone angle of 10°.

attachment. The exact or Taylor-Maccoll solution is tabulated in the Sims tables² and elsewhere. More accurate curve fits than that of Ref. 1 are available in the literature. The expressions presented by Simon and Walter³ and Linnell and Bailey⁴ appear to be superior to the present work. The Linnell-Bailey formula is particularly simple and is amenable to slide rule as well as machine calculation;

$$C_{P_{\text{cone}}} = (4 \sin^2 \theta)(2.5 + 8\beta \sin \theta)/(1 + 16\beta \sin \theta)$$

where $\beta = (M^2 - 1)^{1/2}$ and θ is the half-cone angle. Typical errors using this formula are on the order of 2-3% from shock attachment through high hypersonic Mach numbers.

For purposes of machine computation the following formula developed at Lockheed Missiles & Space Company using least-squares techniques is suggested:

$$C_{P_{\text{cone}}} = (2 \sin^2 \theta)e^x$$

where

$$x = 0.18145 - 0.20923y + 0.09092y^2 + 0.006876y^3 - 0.0062225y^4 - 0.000971y^5$$

and where

$$y = \log_{10}(\beta \sin \theta)$$

The average error is less than 0.75% and the only significant variance occurs near shock detachment. Permissible half-cone angles lie between 2.5° and 30°. Comparative results are presented in Figs. 1 and 2.

Hill's paper presents results at Mach 1. No basically hypersonic formulation can be expected to work in this region. Replacing Mach number by the parameter $(M^2 - 1)^{1/2}$ (unified transonic-hypersonic similarity) will improve his results near shock detachment. Once detachment occurs, however, the inviscid flow on the cone surface is entirely subsonic with the sonic line emanating from the shoulder. The pressure on the cone surface is no longer constant and the similarity breaks down. Characteristically the forecone drag will reach a maximum near shock detachment and will decrease with decreasing Mach number. Several representative curves are shown in the book by Hoerner.⁵

References

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* Aerodynamics Engineer; also Graduate Student, Stanford University, Stanford, Calif. Student Member AIAA.

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Comment on "Drag Coefficient of Small Spherical Particles"

FREDERICK L. SCHUYLER*

General Electric Company, King of Prussia, Pa.

Reply by Author to L. W. Schwartz

J. C. HILL*

Oakland University, Rochester, Mich.

SCHWARTZ raises two criticisms of my recent Technical Note.¹ One is, I think, minor; the other major. First, the minor one; presentation of results at Mach 1 was not intended to imply that Eq. (1) of my Note was valid in the transonic flight regime: it was intended merely to show how well the expression approximated the data in the transonic regime, leaving to the user the technical judgment, dependent on his particular aerodynamic configuration and the accuracy he requires, of how far into the transonic region he may go and still assume that the data of Fig. 1 of my Note represents reality.

Schwartz's major criticism that more exact expressions representing the same data exist is, to my mind, more to the point—and he shows quite clearly that there are such expressions. As I meant to make clear, my expression was developed from Nielsen's² Fig. 9-6, not from the data from which Fig. 9-6 was prepared (which is, apparently, either Ref. 3 or 4; Nielsen does not state which).† The accuracy (or lack thereof) of the present expression is, I believe, primarily because of the graphical nature of the data from which it was developed and because of the limited range of Mach number covered. No attempt was made to improve its accuracy because it was adequate to the purpose for which it was needed—which was to provide order-of-magnitude estimates of drag coefficients for use in control studies, where 5% accuracy would be quite adequate, as opposed to performance studies, where much better accuracy would be desirable. The expressions pointed out by Schwartz apparently provide significantly greater accuracy than my expression.

I am indebted to Mr. Schwartz for bringing these more accurate expressions to my attention. The utility of such expressions for estimating aerodynamic characteristics of complex configurations seems obvious, and a variety of standard expressions for lift, drag, etc. must be in common use by those who deal with such problems on a day-to-day basis.

References

¹ Hill, J. C., "An Empirical Expression for Drag Coefficients of Cones at Supersonic Speeds," *AIAA Journal*, Vol. 7, No. 1, Jan. 1968, pp. 165-167.

² Nielsen, J. N., *Missile Aerodynamics*, McGraw-Hill, New York, 1960, Chap. 9, Sec. 9-4, pp. 275-280.

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* Associate Professor of Engineering.

† Schwartz treats the Sims data as exact; therefore, Schwartz's Lockheed expression and Nielsen's Fig. 9-6 were obtained from different sources. I have not yet been able to compare the two sources; one would hope that this would not be a significant source of error.

THE authors¹ have presented one of the latest experimental studies of small particle acceleration. Additional work, presented in Ref. 2 but not widely distributed, might also be of interest to the authors.

Schuyler carried out both an analytical and experimental study of the trajectory of small (750-1250 μ diam) evaporating liquid droplets. Freon 12 at -22°F was used. Freon droplets were injected into a 12-in. test chamber in which 86°F air moved with a velocity that increased linearly from entrance (50 fps) to exit (200 fps). 6000 frame/sec movie pictures were taken.

Freon 12 was used primarily because of its high evaporation rate at the test conditions. The study was aimed at developing a basic understanding of the dynamics of spray vaporization in the combustion chamber of liquid propellant rockets. The main purpose of the experiments was to check the validity of two major assumptions in the theoretical analysis. These were: the drag coefficient is given by Stoke's law $C_D = 24/Re$, and the droplets' diameter-time variation may be represented by $d^2 = d_0^2 - \lambda t$. Here, d is diameter, d_0 initial diameter, and λ evaporation rate coefficient. Of interest now is the value of C_D , although it should be mentioned that the $D^2 - \lambda t$ law was valid only up to a droplet velocity of 300 fps. Above that, other phenomena take place.

As in the author's study, droplet diameter d , acceleration α , and relative velocity U_R were required to be determined from the experiments. Droplet density ρ' as well as atmospheric density ρ and viscosity μ were assumed constant at -22° and 86°F , respectively. The drag coefficient was calculated as follows:

$$C_D = F_D / \frac{1}{2} \rho A U_R^2$$

$$F_D = \rho' \alpha (\pi d^3 / 6), \quad A = \pi d^2 / 4$$

$$\alpha = \Delta V / \Delta t = (V / \Delta x) \Delta V, \quad Re = \rho U_R d / \mu$$

Therefore,

$$C_D = \{ \frac{4}{3} (\rho' / \mu) d^2 \alpha / U_R \} / Re$$

Measured during the experiments were, droplet position and time (relative to a length scale placed in the test chamber) and local test chamber pressure (from which local air velocity was calculated). Droplet diameter was determined from the photographic records. Each frame of interest was magnified and the droplet measured in four directions: horizontal, vertical, and two diagonals. To determine droplet velocity and acceleration, a curve was passed through the droplet's position-time data. This curve was then differentiated using local finite differences to obtain velocity and acceleration. Though individual large errors probably occurred in this manner, it is expected that the number of data points were sufficiently large to yield reasonably correct values of droplet velocity and acceleration. The results of these experiments

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* Consulting Scientist, Space Sciences Laboratory. Member AIAA.